Factorization in Haar system Hardy spaces

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A Haar system Hardy space is the completion of the linear span of the Haar system $(h_I)_I$, either under a rearrangement-invariant norm $\|\cdot\|$ or under the associated square function norm given by

$$\Big\|\sum_I a_I h_I\Big\|_* = \Big\|\Big(\sum_I a_I^2 h_I^2\Big)^{1/2}\Big\|.$$

Apart from L^p, $1 \leq p < \infty$, the class of these spaces includes all separable rearrangement-invariant function spaces on $[0,1]$ and also the dyadic Hardy space H^1.

Using a unified and systematic approach, we prove that every Haar system Hardy space Y with $Y \neq C(\Delta)$ (where $C(\Delta)$ denotes the space of continuous functions on the Cantor set) has the following property: For every bounded linear operator T on Y, the identity I_Y factors either through T or through $I_Y - T$. Moreover, we show that if T has large diagonal with respect to the Haar system, then the identity factors through T.

Finally, we establish analogous factorization results for the spaces $\ell^p(Y)$, $1 \leq p < \infty$, and we use Pełczyński's decomposition method to show that these spaces are primary.

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