A 2D/1D Multiscale FEM Model for the Nonlinear Eddy Current Problem in Thin Sheets

Markus Schöbinger[1], Joachim Schöberl[1], Paavo Rasilo[2], Karl Hollaus[1]

[1]Technische Universität Wien, Vienna, Austria

[2]Tampere University of Technology, Tampere, Finland

markus.schoebinger@tuwien.ac.at

***Abstract*—A 2D/1D method for the nonlinear eddy current problem in a thin iron sheet is presented. The aim of this work is to reduce the three dimensional problem to a two dimensional one, utilizing ideas from the multiscale finite element method. This allows for a great reduction of computational costs. The method is developed for the $T-\Phi$ formulation and studied in a numerical example.**

***Index Terms*—2D/1D method, eddy current losses, multiscale finite element method, nonlinear problem**

I. Introduction

The main idea of the 2D/1D method is to separate the z dependent components of the solution (using the coordinate system in Fig. 1) and approximate them using predefined shape functions, leaving only two dimensional components as unknowns. A similar approach has been presented in [1], where trigonometric functions in z were used. This contribution is based on polynomial shape functions, which allow the treatment of an air gap or insulation layer and also ensure that the simulated current loops are closed.

II. The Method

The three dimensional reference problem is given as: Find the current vector potential $\mathbf{T} \in H(\text{curl}, \Omega_C)$, with Ω_C being the conducting domain, and the magnetic scalar potential $\Phi \in H^1$ so that

$$\int_\Omega \rho \,\text{curl}\,\mathbf{T}\,\text{curl}\,\mathbf{v} + \frac{\partial}{\partial t}\mu(\mathbf{T}-\nabla\Phi)\cdot(\mathbf{v}-\nabla q)\,d\Omega = 0 \quad (1)$$

for all test functions $\mathbf{v} \in H_0(\text{curl}, \Omega_C)$, $q \in H_0^1$, with μ depending on the solution in a nonlinear way.

The idea of the 2D/1D method is to approximate the solution as an expansion of the form

$$\mathbf{T} - \nabla\Phi \approx \begin{pmatrix} \mathbf{T}_0 + \mathbf{T}_2\phi_2 + \mathbf{T}_4\phi_4 + \mathbf{T}_6\phi_6 + \dots \\ 0 \end{pmatrix} \quad (2)$$

with ϕ_i being the MSFEM shape functions presented in [2]. The $\mathbf{T}_i \in H(\text{curl}, \Omega_{2D})$ are two dimensional unknown functions on Ω_{2D}, the cross section of the iron sheet at $z = 0$.

To derive the 2D/1D weak formulation, the expansion (2) is truncated to a given number of terms and substituted in (1). The integration domain is reduced to Ω_{2D} by averaging the z-dependent coefficients. For nonlinear materials these averages have to be computed for each two dimensional integration point separately and reevaluated in each time step.

The nonlinear system arising in each time step is solved using the techniques presented in [3].

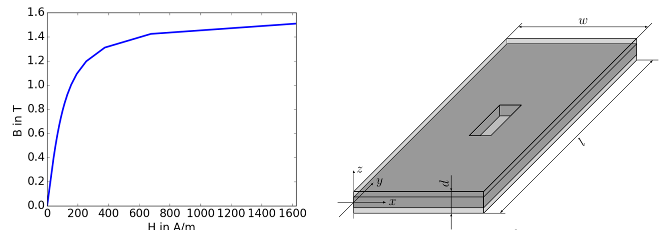

Fig. 1. The dimensions of the sheet are given as $l = 30\,\text{mm}$, $w = 6\,\text{mm}$, $d = 0.5\,\text{mm}$ (5% of which are the air gap). The dimensions of the hole are $1.2 \times 3\,\text{mm}$. The electric conductivity σ is equal to $2.08 \cdot 10^6$ S/m.

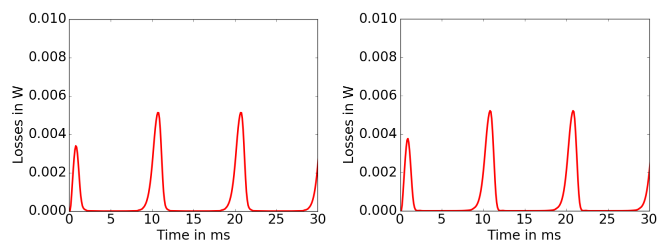

Fig. 2. The calculated eddy current losses over time obtained from the reference problem (left) and the 2D/1D method (right).

III. A Numerical Example

Consider a rectangular iron sheet featuring a rectangular hole. Its geometry and the used B-H curve are given in Fig. 1. The problem is driven by time-harmonic boundary conditions, which ensure a magnetic flux along the y direction.

Both the reference solution and the 2D/1D solution use finite elements of first order. For the 2D/1D solution, the expansion (2) is cut off after the sixth order term $\mathbf{T}_6\phi_6$. It can be seen in Fig. 2, that the 2D/1D method gives a good approximation of the losses for all times.

IV. Acknowledgment

This work was supported by the Austrian Science Fund (FWF) under Project P 27028 and the Academy of Finland.

References

[1] P. Rasilo, E. Dlala, K. Fonteyn, J. Pippuri, A. Belahcen, and A. Arkkio, "Model of laminated ferromagnetic cores for loss prediction in electrical machines," *IET Electric Power Applications*, vol. 5, no. 7, pp. 580–588, August 2011.

[2] K. Hollaus and J. Schöberl, "Some 2-d multiscale finite-element formulations for the eddy current problem in iron laminates," *IEEE Transactions on Magnetics*, vol. 54, no. 4, pp. 1–16, April 2018.

[3] O. Bíró and K. Preis, "Finite element calculation of time-periodic 3d eddy currents in nonlinear media," in *Advanced Computational Electromagnetics*, T. Homna, Ed. Budapest, Hungary: Elsevier, 1995, pp. 62–74.