An Equilibrated Error Estimator for the Multiscale FEM of a Linear Eddy Current Problem

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***Abstract*—The multiscale finite element method (MSFEM) is an efficient way to solve the eddy current problem on laminated iron cores without the necessity to resolve each laminate by the mesh. This paper presents an a posteriori error estimator for a MSFEM using the T-formulation of the eddy current problem. The estimator is based on flux equilibration and gives an upper bound for the error without generic constants. It requires the solution of small local problems on the fine scale.**

***Index Terms*—Eddy currents, Multiscale, Finite Element Method, Error Estimator**

I. The Multiscale Finite Element Method

Consider the linear eddy current problem using the current vector potential $\mathbf{T}$. If the closing of the field at the ends of the iron core is ignored, the assumption $\mathbf{T} = u\mathbf{e}_z$ holds and the problem can be reduced to two dimensions with the weak formulation: Find $u \in H^1(\Omega), u = u_D$ on $\partial\Omega$, so that

$$\int \rho \nabla u \cdot \nabla v + i\omega\mu u \cdot v \, d\Omega = 0 \qquad (1)$$

for all $v \in H_0^1(\Omega)$. The parameters in (1) are the electric resistivity ρ, the magnetic permeability μ, the angular frequency ω and the imaginary unit i.

In [1] the multiscale approximation

$$u = u_0 + \phi_2 u_2 \qquad (2)$$

is proposed. Here $u_0, u_2 \in H^1(\Omega)$ are defined on a coarse mesh which does not resolve the single laminates. The fine scale behavior of the solution is provided by the micro-shape function ϕ_2, which is a quadratic polynomial on each laminate.

The ansatz (2) is used in (1) for both the trial function and the test function. The material coefficients and ϕ_2 are averaged over one period width, which gives a problem that is posed only on the coarse scale. For details, see [1].

II. The Error Estimator

The estimator is based on the techniques presented in [2], which use the Theorem of Prager and Synge. For the problem $-\operatorname{div} \rho \nabla u = f$ it states that for the finite element solution u_h and each admissible flux $\sigma \in H(\operatorname{div})$

$$\|\nabla u - \nabla u_h\|_\rho \leq \|\rho \nabla u_h - \sigma\|_\rho \qquad (3)$$

holds, where $\|.\|_\rho$ denotes the energy norm. The method constructs a flux σ so that the estimator $\rho \nabla u_h - \sigma$ can be split into local components that can be calculated in parallel.

For the given problem (1) a relation can be derived which is similar to (3) up to higher order terms, i.e. it also gives an error estimatior without a generic constant.

In [3] a similar approach has been used to estimate the error of the MSFEM solution with respect to the continuous multiscale solution. In this paper the more practically relevant error with respect to the solution of the original problem is measured by constructing the components of the equilibrated flux on local subdomains of the fine mesh.

III. Numerical Example

Consider a quadratic iron core with 10 laminates, each $0.6mm$ thick, with a fill factor of 0.9. The material parameters are $\mu = 10,000\mu_0$ and $\rho = 5 \cdot 10^{-7} m/S$ in iron.

The MSFEM solution is calculated on a coarse mesh containing only 25 elements. For comparison, a reference solution is calculated on a fine mesh with $1,395$ elements, which accurately resolves all laminates. In Fig. 1 the error with respect to the reference solution and the error estimator are shown on the reference mesh. It can be seen that the estimator gives a satisfactory approximation of the real error.

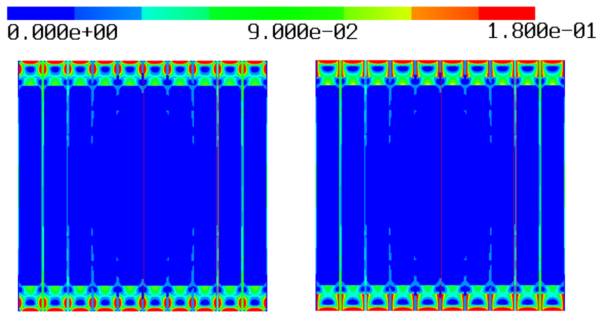

Fig. 1. The energy error of the multiscale solution with respect to the reference solution (left) and the error estimator (right).

IV. Acknowledgment

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References

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