A Hierarchical Error Estimator for the MSFEM for the Eddy Current Problem in 3D

Markus Schöbinger[1], Joachim Schöberl[1] and Karl Hollaus[1]

[1]Institute for Analysis and Scientific Computing TU Wien, Wiedner Hauptstraße 8-10, A-1040 Vienna, Austria
E-mail: markus.schoebinger@tuwien.ac.at

***Abstract*—The eddy current problem is solved on a stack of iron sheets using the multiscale finite element method (MSFEM). This allows a calculation of the solution without having to resolve each sheet in the finite element mesh. In order to further increase the efficiency of the multiscale method, a hierarchical error estimator is implemented, which allows for p-refinement on the chosen mesh. The estimator is based on the solution of small local problems posed on the space of the higher order finite element bubble functions.**

***Index Terms*—Eddy currents, multiscale FEM, hierarchical error estimator**

I. The Multiscale Method

The weak formulation of the linear eddy current problem in the frequency domain is given as: Find the magnetic vector potential $\mathbf{A} \in H(\text{curl})$, so that

$$\underbrace{\int_\Omega \mu^{-1}\text{curl}\mathbf{A}\cdot\text{curl}\mathbf{v} + i\omega\sigma\mathbf{A}\cdot\mathbf{v}\,\mathrm{d}\Omega}_{:=a(\mathbf{A},\mathbf{v})} = \underbrace{\int_\Omega \mathbf{J}\cdot\mathbf{v}\,\mathrm{d}\Omega}_{:=f(\mathbf{v})} \tag{1}$$

for all $\mathbf{v} \in H(\text{curl})$ with the magnetic permeability μ, the electric conductivity σ and the angular frequency ω.

The first order MSFEM approximates the magnetic vector potential via the expansion

$$\mathbf{A}_{MS} = \mathbf{A}_0 + \phi_1\mathbf{A}_1 + \nabla(\phi_1 w_1) \tag{2}$$

with the unknown components $\mathbf{A}_0, \mathbf{A}_1 \in H(\text{curl})$ and $w_1 \in H^1$ defined on a coarse mesh which does not resolve the iron sheets. The local behavior is resolved by the piecewise polynomial shape function ϕ_1.

Using (2) in (1) gives the new problem

$$\bar{a}(\mathbf{A}_{MS}, \mathbf{v}_{MS}) = \bar{f}(\mathbf{v}_{MS}). \tag{3}$$

Details about the method and the averaged forms $\bar{a}$ and $\bar{f}$ can be found in [1].

II. The Error Estimator

The developed estimator is based on the known hierarchical estimators for the H^1 (see for example [2]) and the $H(\text{curl})$ (see [3]), which are extended to fit into the multiscale setting.

Let $\mathcal{V}^n := \mathcal{N}^n \times \mathcal{N}^n \times \mathcal{P}^{n+1}$ with the n^{th} order Nedelec space $\mathcal{N}^n \subset H(\text{curl})$ and the space of continuous element-wise polynimals of order $n+1$, denoted by $\mathcal{P}^{n+1} \subset H^1$. Let $\mathbf{A}^n_{MS} \in \mathcal{V}^n$ be the finite element solution of (3). Define the space of higher order bubble functions $\mathcal{B}^n := \mathcal{V}^{n+1} \setminus \mathcal{V}^n$. The estimating function is given as the solution of: Find $\mathbf{z} \in \mathcal{B}^n$ so that

$$\bar{a}(\mathbf{z}, \chi) = \bar{a}(\mathbf{A}^n_{MS}, \chi) - \bar{f}(\chi) \qquad \forall \chi \in \mathcal{B}^n. \tag{4}$$

III. Numerical Example

Consider an eight of a stack of iron sheets, as illustrated in Fig.1. The error estimator was applied to each finite element individually. In each iteration all elements with an estimated error greater than one fourth of the maximum over all elements were refined. It can be seen that the estimator correctly refines in the regions of the greatest variation and the adaptive method is much more efficient with respect to the degrees of freedom, compared to uniform refinement.

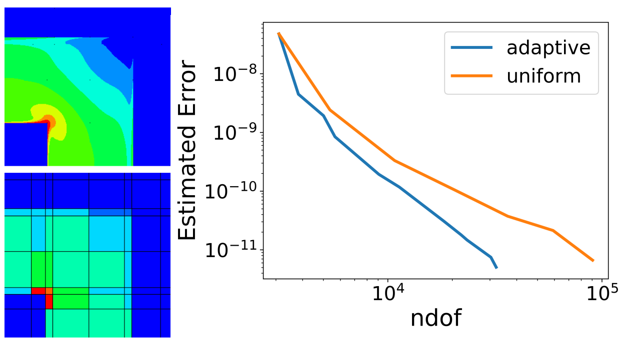

Fig. 1. The current density J in a cross section of the domain (top left), the distribution of refinements in each element (bottom left, adpative orders ranging from 0 (dark blue) to 9 (dark red)) and the estimated total error with respect to the degrees of freedom (right).

IV. Acknowledgment

This work was supported by the Austrian Science Fund (FWF) under Project P 31926.

References

[1] K. Hollaus, J. Schöberl and M. Schöbinger "The Multiscale Finite Element Method in three dimensions to simulate the Eddy Current Problem in Laminated Iron Cores," submitted to COMPEL, 2018

[2] R. Verfürth, "A Review of A Posteriori Error Estimation and Adaptive MeshRefinement Techniques," WileyTeubner, Chichester, Stuttgart, 1996.

[3] R. Beck, R. Hiptmair and B. I. Wohlmuth, "Hierarchical Error Estimator for Eddy Current Computation," In ENUMATH99: Proceedings of the 3rd European Conference on Numerical Mathematics and Advanced Applictions, 1999