MSFEM for a 2D1D Eddy Current Model Including Edge Effects

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Overview

- Main Ideas

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- 2D1D for the A, V - Formulation
- 2D1D for the T, Φ - Formulation

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- Numerical Results

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- Numerical Results
- Conclusions

Main Ideas

In a rotating machine, each steel sheet is exposed to roughly the same field.

$\Rightarrow$ It suffices to simulate a single sheet.

$\Rightarrow$ Already a great reduction of computation cost.

Problems:

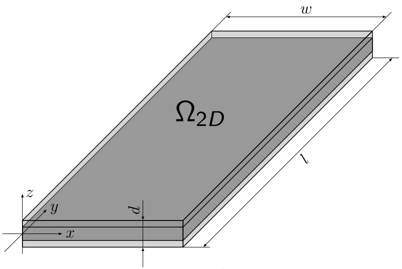
- Resolving the penetration depth requires a fine mesh at all boundaries.
- Thickness is small compared to the other dimensions.
- Air gap between sheets is small even compared to the thickness.

$\Rightarrow$ Solving in three dimensions is expensive.

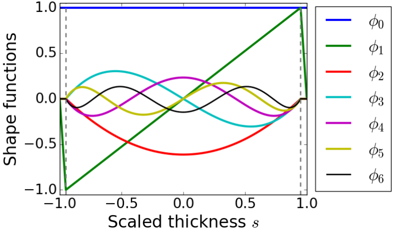
Main Ideas

Expand the 3D solution u in the form:

$$u(x, y, z) \approx \sum_i u_i(x, y)\phi_i(z)$$



$$\Omega = \Omega_{2D} \times [-\tfrac{d}{2}, \tfrac{d}{2}]$$



- The shape functions are piecewise polynomials to treat the air gap.
- Gauss-Lobatto polynomials are used in iron.

The $A - V$ Formulation

We use the magnetic vector potential $\mathbf{A} \in H(\mathrm{curl})$ and the electric scalar potential $V \in H^1$, satisfying

$$\begin{aligned} \operatorname{curl} \mu^{-1} \operatorname{curl} \mathbf{A} + i\omega\sigma(\mathbf{A} - \nabla V) &= \mathbf{0} \\ \operatorname{div} i\omega\sigma(\mathbf{A} - \nabla V) &= 0 \end{aligned}$$

using. . .

μ . . . the magnetic permeability
σ . . . the electric conductivity
ω . . . the angular frequency

The $A - V$ Formulation, reference problem

$$\operatorname{curl} \mu^{-1} \operatorname{curl} \mathbf{A} + i\omega\sigma(\mathbf{A} - \nabla V) = \mathbf{0} \tag{1}$$

$$\operatorname{div} i\omega\sigma(\mathbf{A} - \nabla V) = \mathbf{0} \tag{2}$$

Multiply (1) with $\mathbf{v} \in H(\text{curl})$, (2) with $q \in H^1$.

Passing to the weak formulation:

Find $\mathbf{A} \in H(\text{curl})$, $V \in H^1$ so that

$$\int_\Omega \mu^{-1} \operatorname{curl} \mathbf{A} \cdot \operatorname{curl} \mathbf{v} + i\omega\sigma(\mathbf{A} - \nabla V) \cdot (\mathbf{v} - \nabla q) \, d\Omega = 0$$

for all $\mathbf{v} \in H(\text{curl})$, $q \in H^1$.

Boundary conditions depend on the given problem.

The 2D1D approach

Disregarding edge effects, $\mathbf{A} - \nabla V$ behaves as an odd function in z.

This motivates the ansatz

$$\mathbf{A} - \nabla V \approx \begin{pmatrix} \phi_1(z)\mathbf{A_1}(x,y) + \phi_3(z)\mathbf{A_3}(x,y) + \phi_5(z)\mathbf{A_5}(x,y) + \dots \\ 0 \end{pmatrix}$$

with $\mathbf{A_1}, \mathbf{A_3}, \mathbf{A_5}, \dots \in H(\mathrm{curl}, \Omega_{2D})$.

The 2D1D approach

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with $\mathbf{A_1}, \mathbf{A_3}, \mathbf{A_5}, \dots \in H(\text{curl}, \Omega_{2D})$.

How to translate 3D boundary conditions to 2D?

The Unknown $\mathbf{A_1}$

The first term has a special role:

$$\int \phi_{1,z}\, dz \neq 0, \quad \int \phi_{1,z}\, dz = 0, i > 1$$

$\Rightarrow$ The first order term controls the total magnetic flux.

The higher order terms act as correctors without changing the total flux.

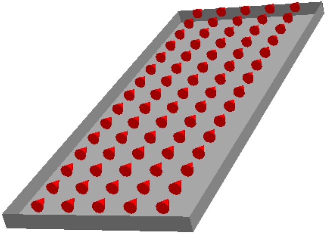
$\mathbf{A_1}$ is either obtained from a physical model or by introducing a new scalar potential:

$$\mathbf{A_1} = \nabla u$$

The Unknown $\mathbf{A_1}$: An Example

Assume the steel sheet to be aligned with the coordinate axes and Φ_B a given magnetic flux through the cross section S. We calculate:

$$
\begin{aligned}
\Phi_B &= \int_S \operatorname{curl} A \cdot dS \\
&= \int_0^w \int_{-\frac{d_{Fe}}{2}}^{\frac{d_{Fe}}{2}} \phi_{1,z} u_x \, dz \, dx \\
&= \int_{-\frac{d_{Fe}}{2}}^{\frac{d_{Fe}}{2}} \phi_{1,z} \, dz \int_0^w u_x \, dx \\
&= \int_{-\frac{d_{Fe}}{2}}^{\frac{d_{Fe}}{2}} \phi_{1,z} \, dz (u(w) - u(0))
\end{aligned}
$$



The B field

$\Rightarrow \Phi_B$ directly yields the boundary conditions for u.

Derivation of the 2D1D formulation

Use

$$\underbrace{\mathbf{A}-\nabla V}_{\text{trial function}} = \begin{pmatrix}\phi_1 \nabla u\\ 0\end{pmatrix}, \quad \underbrace{\mathbf{v}-\nabla q}_{\text{test function}} = \begin{pmatrix}\phi_1 \nabla v\\ 0\end{pmatrix}$$

in

$$\int_\Omega \mu^{-1}\,\mathrm{curl}\,\mathbf{A}\cdot\mathrm{curl}\,\mathbf{v} + i\omega\sigma(\mathbf{A}-\nabla V)\cdot(\mathbf{v}-\nabla q)\,d\Omega = 0$$

to get

$$\int_\Omega \mu^{-1}\phi_{1,z}^2 \begin{pmatrix}-u_y\\ u_x\\ 0\end{pmatrix}\cdot\begin{pmatrix}-v_y\\ v_x\\ 0\end{pmatrix} + i\omega\sigma\phi_1^2 \begin{pmatrix}u_x\\ u_y\\ 0\end{pmatrix}\cdot\begin{pmatrix}v_x\\ v_y\\ 0\end{pmatrix}\,d\Omega = 0$$

Derivation of the 2D1D formulation

$$\int_{\Omega_{2D}} \int_{-\frac{d}{2}}^{\frac{d}{2}} \mu^{-1}\phi_{1,z}^2 \begin{pmatrix} -u_y \\ u_x \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -v_y \\ v_x \\ 0 \end{pmatrix} \quad + i\omega\sigma\phi_1^2 \begin{pmatrix} u_x \\ u_y \\ 0 \end{pmatrix} \cdot \begin{pmatrix} v_x \\ v_y \\ 0 \end{pmatrix} \, dz \, d\Omega_{2D} = 0$$

$$\int_{\Omega_{2D}} \nabla u \cdot \nabla v \underbrace{\int_{-\frac{d}{2}}^{\frac{d}{2}} \mu^{-1}\phi_{1,z}^2 \, dz}_{=:\overline{\mu^{-1}\phi_{1,z}^2}} \qquad + i\omega \nabla u \cdot \nabla v \underbrace{\int_{-\frac{d}{2}}^{\frac{d}{2}} \sigma\phi_1^2 \, dz}_{=:\overline{\sigma\phi_1^2}} \, d\Omega_{2D} = 0$$

Final formulation: Find $u \in H^1(\Omega_{2D})$ so that

$$\int_{\Omega_{2D}} (\overline{\mu^{-1}\phi_{1,z}^2} + i\omega\overline{\sigma\phi_1^2}) \nabla u \cdot \nabla v \, d\Omega_{2D} = 0$$

for all $v \in H^1(\Omega_{2D})$.

Derivation of the 2D1D formulation

For smaller penetration depths, higher order terms are needed:

$$\mathbf{A} - \nabla V = \begin{pmatrix} \phi_1 \nabla u + \phi_3 \mathbf{A}_3 + \phi_5 \mathbf{A}_5 \\ 0 \end{pmatrix}$$

$$\mathbf{V} - \nabla q = \begin{pmatrix} \phi_1 \nabla v + \phi_3 \mathbf{v}_3 + \phi_5 \mathbf{v}_5 \\ 0 \end{pmatrix}$$

is used in

$$\int_\Omega \mu^{-1} \operatorname{curl} \mathbf{A} \cdot \operatorname{curl} \mathbf{v} + i\omega\sigma(\mathbf{A} - \nabla V) \cdot (\mathbf{v} - \nabla q)\, d\Omega = 0.$$

The terms in the integrals are separated; similar calculations as above.

Derivation of the 2D1D formulation

Final problem:

Find $u \in H^1(\Omega_{2D})$, including boundary conditions and $\mathbf{A}_3, \mathbf{A}_5 \in H_0(\mathrm{curl}, \Omega_{2_D})$ so that

$$
\begin{aligned}
\int_{\Omega_{2D}} & (\overline{\mu^{-1}\phi_{1,z}^2} + i\omega\overline{\sigma\phi_1^2})\nabla u \cdot \nabla v + \overline{\mu^{-1}\phi_{3,z}^2}\mathbf{A}_3 \cdot \mathbf{v}_3 \\
& + \overline{\mu^{-1}\phi_3^2}\,\mathrm{curl}\,\mathbf{A}_3\,\mathrm{curl}\,\mathbf{v}_3 + \overline{\mu^{-1}\phi_{5,z}^2}\mathbf{A}_5 \cdot \mathbf{v}_5 + \overline{\mu^{-1}\phi_5^2}\,\mathrm{curl}\,\mathbf{A}_5\,\mathrm{curl}\,\mathbf{v}_5 \\
& + \overline{\mu^{-1}\phi_3\phi_5}(\mathrm{curl}\,\mathbf{A}_3\,\mathrm{curl}\,\mathbf{v}_5 + \mathrm{curl}\,\mathbf{A}_5\,\mathrm{curl}\,\mathbf{v}_3) \\
& + i\omega\overline{\sigma\phi_1\phi_3}(\nabla u \cdot \mathbf{v}_3 + \mathbf{A}_3 \cdot \nabla v) + i\omega\left(\overline{\sigma\phi_3^2}\mathbf{A}_3 \cdot \mathbf{v}_3 + \overline{\sigma\phi_5^2}\mathbf{A}_5 \cdot \mathbf{v}_5\right) \\
& + i\omega\left(\overline{\sigma\phi_3\phi_5}(\mathbf{A}_3 \cdot \mathbf{v}_5 + \mathbf{A}_5 \cdot \mathbf{v}_3)\right)\, d\Omega_{2D} = 0
\end{aligned}
$$

for all $v \in H_0^1(\Omega_{2D})$, $\mathbf{v}_3, \mathbf{v}_5 \in H_0(\mathrm{curl}\,\Omega_{2D})$.

$T - \Phi$ Formulation

We use the current vector potential $\mathbf{T} \in H(\text{curl})$ and the magnetic scalar potential $\Phi \in H^1$, satisfying

$$\begin{aligned} \operatorname{curl} \rho \operatorname{curl} \mathbf{T} + i\omega\mu(\mathbf{T} - \nabla\Phi) &= \mathbf{0} \\ \operatorname{div} i\omega\mu(\mathbf{T} - \nabla\Phi) &= 0 \end{aligned}$$

using...

$\rho \ldots$ the electric resistivity
$\mu \ldots$ the magnetic permeability
$\omega \ldots$ the angular frequency

The $T - \Phi$ Formulation, reference solution

$$\operatorname{curl} \rho \operatorname{curl} \mathbf{T} + i\omega\mu(\mathbf{T} - \nabla\Phi) = \mathbf{0} \tag{3}$$

$$\operatorname{div} i\omega\mu(\mathbf{T} - \nabla\Phi) = \mathbf{0} \tag{4}$$

Multiply (3) with $\mathbf{v} \in H(\text{curl})$, (4) with $q \in H^1$.

Passing to the weak formulation:

Find $\mathbf{T} \in H(\text{curl})$, $\Phi \in H^1$ so that

$$\int_\Omega \rho \operatorname{curl} \mathbf{T} \cdot \operatorname{curl} \mathbf{v} + i\omega\mu(\mathbf{T} - \nabla\Phi) \cdot (\mathbf{v} - \nabla q)\, d\Omega = 0$$

for all $\mathbf{v} \in H(\text{curl})$, $q \in H^1$.

Boundary conditions depend on the given problem.

The 2D1D approach

Expansion via even polynomials:

$$\mathbf{T} - \nabla\Phi \approx \begin{pmatrix} \phi_0(z)\mathbf{T_0}(x,y) + \phi_2(z)\mathbf{T_2}(x,y) + \phi_4(z)\mathbf{T_4}(x,y) + \dots \\ 0 \end{pmatrix}$$

with $\mathbf{T_0}, \mathbf{T_2}, \mathbf{T_4}, \dots \in H(\mathrm{curl}, \Omega_{2D})$.

Boundary conditions are obtained from the reference problem via

$$\mathbf{T_0} = \nabla u.$$

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Expansion via even polynomials:

$$\mathbf{T} - \nabla\Phi \approx \begin{pmatrix} \phi_0(z)\mathbf{T_0}(x,y) + \phi_2(z)\mathbf{T_2}(x,y) + \phi_4(z)\mathbf{T_4}(x,y) + \dots \\ 0 \end{pmatrix}$$

with $\mathbf{T_0}, \mathbf{T_2}, \mathbf{T_4}, \dots \in H(\text{curl}, \Omega_{2D})$.

Boundary conditions are obtained from the reference problem via

$$\mathbf{T_0} = \nabla u.$$

Important difference: $\mathbf{T} - \nabla\Phi$ behaves as an even function in z everywhere.

$\Rightarrow$ Edge effects are treated correctly automatically.

Derivation of the 2D1D formulation

Similar to before:

$$\mathbf{T} - \nabla\Phi = \begin{pmatrix} \phi_0 \nabla u + \phi_2 \mathbf{T}_2 + \dots \\ 0 \end{pmatrix}, \mathbf{V} - \nabla q = \begin{pmatrix} \phi_0 \nabla v + \phi_2 \mathbf{v}_2 + \dots \\ 0 \end{pmatrix}$$

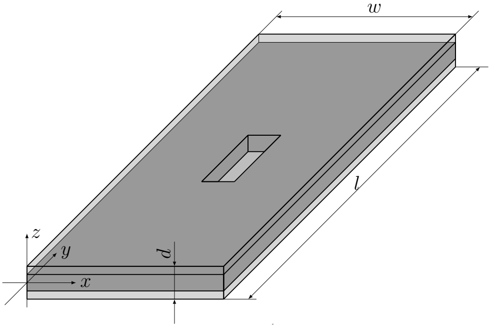
is used in

$$\int_\Omega \rho \operatorname{curl} \mathbf{T} \cdot \operatorname{curl} \mathbf{v} + i\omega\mu(\mathbf{T} - \nabla\Phi) \cdot (\mathbf{v} - \nabla q)\, d\Omega = 0$$

which results, after separation of integrals, in the 2D system.

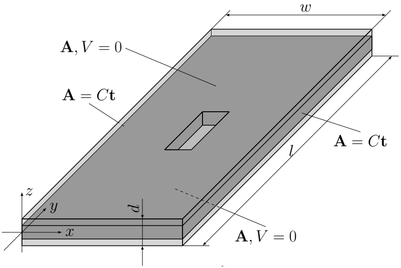
Boundary conditions are included by taking the lowest order term as a gradient.

Example: Rectangular Sheet with a Hole

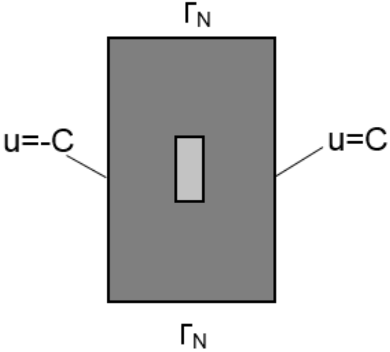


- $l = 30$ mm
- $w = 6$ mm
- $d = 0.5$ mm
- Fillfactor $= 95\%$
- $\mu = 1,000\mu_0$
- $\sigma = 2.08 \cdot 10^6\ S/m$

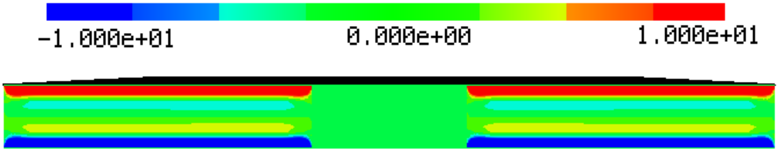
Example: Rectangular Sheet with a Hole, Boundary Conditions A, V - Formulation



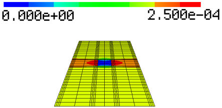
reference solution



first term 2D1D

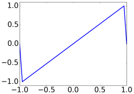


Reference solution, $\mathbf{A} - \nabla V$ (real part of x-component)



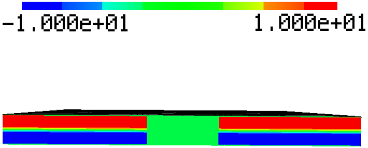
A_1

$\times$

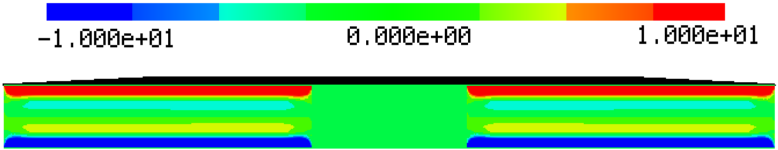


ϕ_1

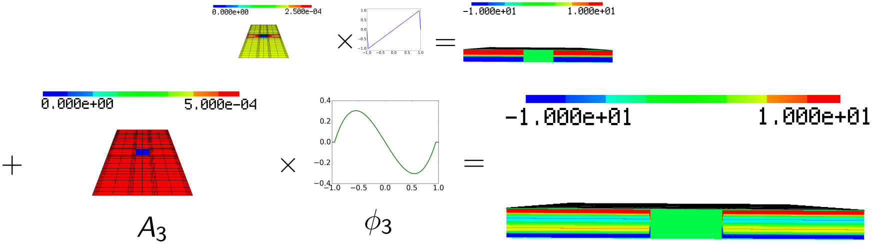
$=$



Example: Rectangular Sheet with a Hole, $f = 30$ kHz



Reference solution, $\mathbf{A} - \nabla V$ (real part of x-component)



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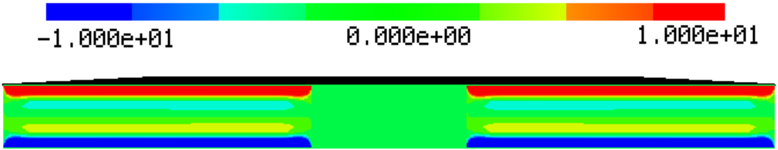
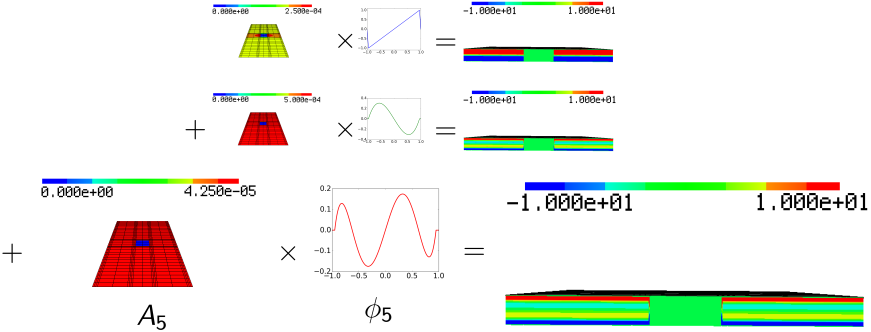
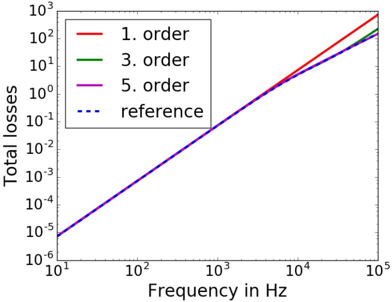


Reference solution, $\mathbf{A} - \nabla V$ (real part of x-component)



Example: Rectangular Sheet with a Hole, $f = 30$ kHz

Comparison of the eddy current losses over a large frequency range:



- Expected behavior:
  $P = C * f^2$ for low f
  $P = C * f^{1.5}$ for high f.
- Each expansion works well up to a "limit frequency".
- First order expansion continues quadratically.

Example: Rectangular Sheet with a Hole, $f = 30$ kHz

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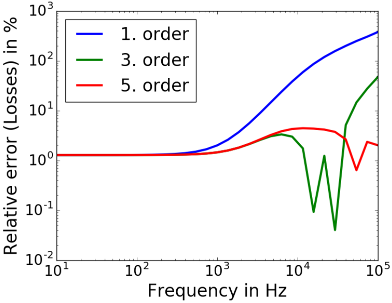
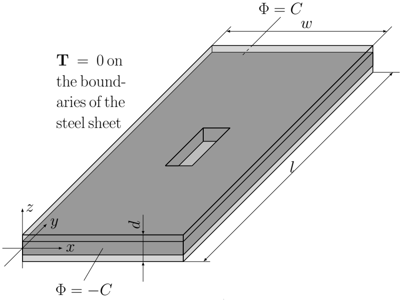
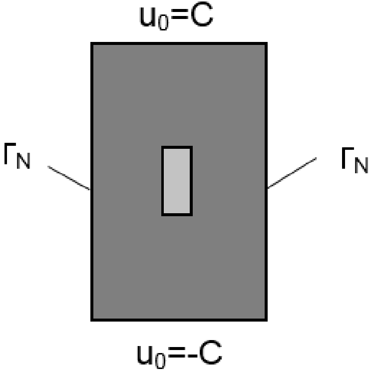


- For low frequencies, a low number of terms suffices.
- Each expansion works well up to a "limit frequency".
- A "base error" of 1% because of the edge effects.

Example: Rectangular Sheet with a Hole, Boundary Conditions T, Φ - Formulation

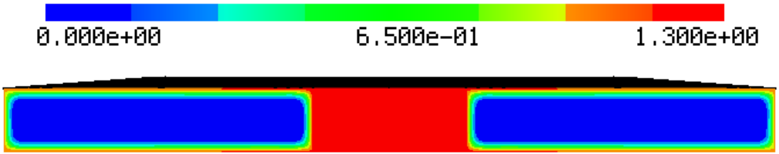


reference solution

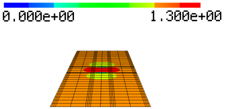


first term 2D1D

Example: Rectangular Sheet with a Hole, $f = 30$ kHz

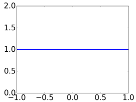


Reference solution, $\mathbf{T} - \nabla\Phi$ (real part of y-component)



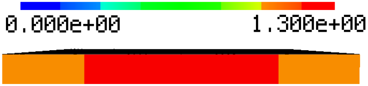
T_0

$\times$

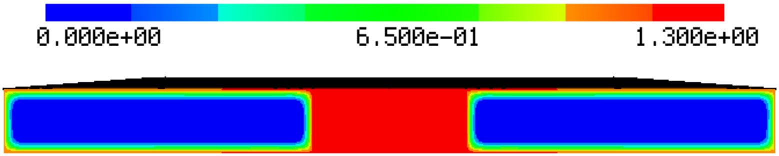


ϕ_0

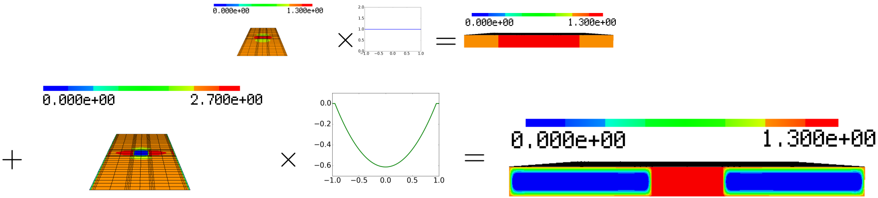
$=$



Example: Rectangular Sheet with a Hole, $f = 30$ kHz

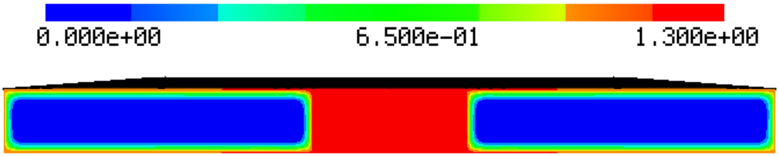


Reference solution, $\mathbf{T} - \nabla\Phi$ (real part of y-component)

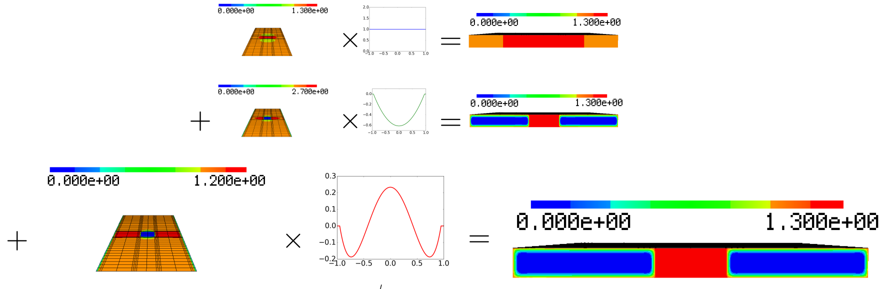


T_2 ϕ_2

Example: Rectangular Sheet with a Hole, $f = 30$ kHz



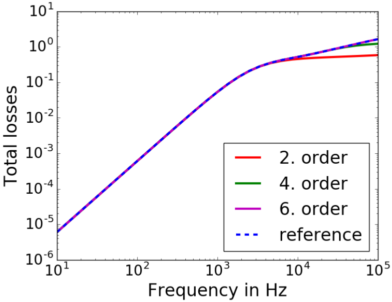
Reference solution, $\mathbf{T} - \nabla\Phi$ (real part of y-component)



T_4 ϕ_4

Example: Rectangular Sheet with a Hole, $f = 30$ kHz

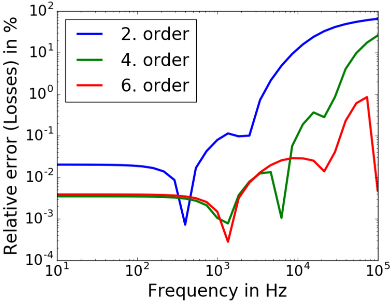
Comparison of the eddy current losses over a large frequency range:



- For low frequencies, a low number of terms suffices.
- Higher frequencies require higher number of terms.
- Each expansion works nearly perfectly up to a “limit frequency”.
- Three terms seem to suffice for the used frequencies.

Example: Rectangular Sheet with a Hole, $f = 30$ kHz

Comparison of the eddy current losses over a large frequency range:



- For low frequencies, a low number of terms suffices.
- Higher frequencies require higher number of terms.
- Each expansion works nearly perfectly up to a "limit frequency".
- The error stays below 1% using three terms.

Conclusions

- Two 2D1D formulations for the eddy current problem have been presented.

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- Using enough terms, the method works well over a high range of frequencies.

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- Using the current vector potential, the edge effects can be resolved.

Conclusions

- Two 2D1D formulations for the eddy current problem have been presented.
- Using enough terms, the method works well over a high range of frequencies.
- Using the current vector potential, the edge effects can be resolved.
- To do: Considering nonlinear materials and hysteresis.

Thank you for your attention!