MSFEM with Network Coupling for the Eddy Current Problem of a Toroidal Transformer

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Overview

- Problem Setting
- Coupling with Network
- Multiscale Ansatz
- Dealing with the Nonlinearity
- Numeric Results
- Conclusion and Outlook

Problem Setting

Given the voltage $u(t) = U_{max}\cos(2\pi f t)$, find the current $i(t) \in \mathbb{R}$, $A(t) \in H(\text{curl})$ so that

$$\int_\Omega \nu(A)\,\text{curl}\,A\,\text{curl}\,v\,d\Omega + \frac{\partial}{\partial t}\int_\Omega \sigma A v\,d\Omega = \int_{\Gamma(\Omega)} K v\,d\Gamma$$

$$-\frac{\partial \Phi}{\partial t} + iRj = u$$

for all $v \in H(\text{curl})$ and $j \in \mathbb{R}$.

$\nu \ldots$ inverse of the magnetic permeability
$\sigma \ldots$ electric conductivity
$\Phi \ldots$ magnetic flux density
$K \ldots$ surface current density
$R \ldots$ electric resistor

Network Coupling

With the following manipulations:

$$\int_{\Gamma(\Omega)} Kv \, d\Gamma = i \int_{\Gamma(\Omega)} \tau v \, d\Gamma$$

$$K = i \frac{N}{2\pi r} \quad \tau = \frac{N}{2\pi r}$$

$\tau \ldots$ turn density
$N \ldots$ No. windings
$r \ldots$ radius

$$-\frac{\partial \Phi}{\partial t} = -\int_S \frac{\partial A}{\partial t} \, ds = -N \int_{l_W} \frac{\partial A}{\partial t} \frac{2\pi r}{2\pi r} \, ds = -\frac{\partial}{\partial t} \int_{\Gamma(\Omega)} A\tau \, d\Gamma$$

$l_w \ldots$ length of one winding

Problem Setting

Given the voltage $u(t) = U_{max}\cos(2\pi f t)$, find the current $i(t) \in \mathbb{R}$, $A(t) \in H(\text{curl})$ so that

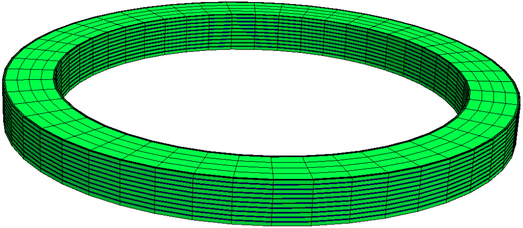
$$\int_\Omega \nu(A)\,\text{curl}\,A\,\text{curl}\,v\,d\Omega + \frac{\partial}{\partial t}\int_\Omega \sigma A v\,d\Omega - i\int_{\Gamma(\Omega)} \tau v\,d\Gamma = 0$$

$$-j\frac{\partial}{\partial t}\int_{\Gamma(\Omega)} A\tau\,d\Gamma + iRj = u$$

for all $v \in H(\text{curl})$ and $j \in \mathbb{R}$.

$\nu\ldots$ inverse of the magnetic permeability
$\sigma\ldots$ electric conductivity
$\tau\ldots$ turn density $R\ldots$ electric resistor

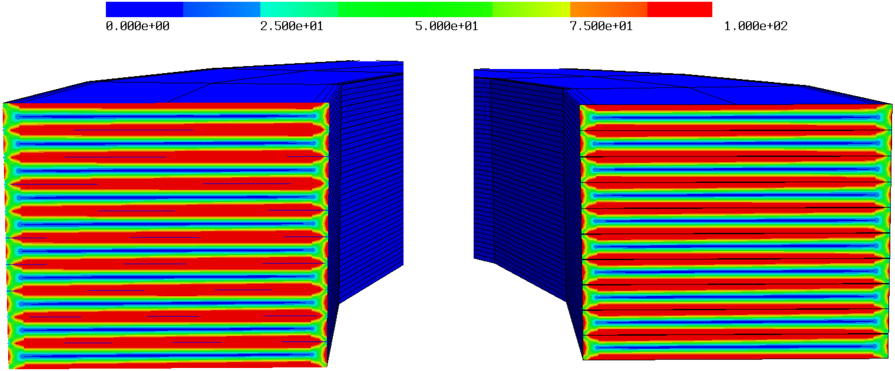
The Domain



- outer radius: 30mm
- inner radius: 24mm
- height: 5mm
- 10 iron sheets
- primary windings: 75
- 1% air gap

Measurements provided by the RWTH Aachen

Solution Schematic



Absolute value of $J = \sigma A$ (linear, time-harmonic)
left and right cross section of the core

Problem and solution are independent of angle
$\Rightarrow$ Reduction to 2D is possible

Reduction to 2D

The assumed rotational symmetry allows for a simplification using cylindrical coordinates.

	full model	simplified
A	$\begin{pmatrix} A_r(r,\varphi,z) \\ A_\varphi(r,\varphi,z) \\ A_z(r,\varphi,z) \end{pmatrix}$	$\begin{pmatrix} A_r(r,z) \\ 0 \\ A_z(r,z) \end{pmatrix}$
$\operatorname{curl} A$	$\begin{pmatrix} \frac{1}{r}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z} \\ \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \\ \frac{1}{r}\left(\frac{\partial r A_\varphi}{\partial r} - \frac{\partial A_r}{\partial \varphi}\right) \end{pmatrix}$	$\begin{pmatrix} 0 \\ \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \\ 0 \end{pmatrix}$

This leads to an analogous formulation in 2D using the 2D curl.

Motivation for Multiscale

The solution shows different behavior at different scales:

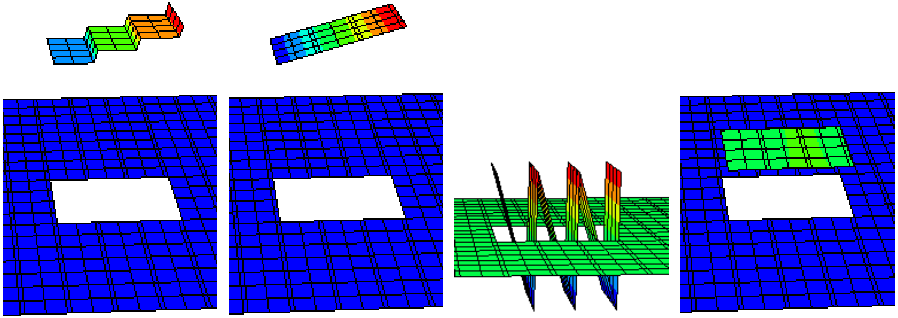
- There is a large variation in each iron sheet (and air gap).
- There is a small variation from sheet to sheet.

The chance in the global behavior can be discretized on a very coarse mesh.

However, a fine mesh is needed to capture the local behavior.

Idea of the Multiscale Method

Split the function into the slowly and the rapidly changing parts:



$$A_h = A_{h,0} + \phi \cdot A_{h,1}$$

The Multiscale Method

We propose the ansatz

$$A(r,z) = A_0(r,z) + \phi(z)\begin{pmatrix} A_1(r,z) \\ 0 \end{pmatrix} + \nabla(\phi(z)w(r,z))$$

$$\operatorname{curl} A(r,z) = \operatorname{curl} A_0(r,z) - \frac{\partial \phi}{\partial z}(z)A_1(r,z)$$

with $A_0 \in H(\mathrm{curl})$, $A_1 \in L^2$ and $w \in H^1$.

Note that the equations use the 2D curl

$$\operatorname{curl}\begin{pmatrix} A_r \\ A_z \end{pmatrix} = \frac{\partial A_z}{\partial r} - \frac{\partial A_r}{\partial z}$$

which is a scalar function.

The Multiscale Method

In the equation system

$$\int_\Omega \nu(A)\,\mathrm{curl}\,A\,\mathrm{curl}\,v\,d\Omega + \frac{\partial}{\partial t}\int_\Omega \sigma A v\,d\Omega - i\int_{\Gamma(\Omega)} \tau v\,d\Gamma = 0$$

$$-\frac{\partial}{\partial t}\int_{\Gamma(\Omega)} A\tau j\,d\Gamma + iRj = uj.$$

perform the substitutions

$$\begin{aligned} A &\quad\rightarrow\quad A_0 + \phi\begin{pmatrix} A_1 \\ 0 \end{pmatrix} + \nabla(\phi w) \\ v &\quad\rightarrow\quad v_0 + \phi\begin{pmatrix} v_1 \\ 0 \end{pmatrix} + \nabla(\phi q). \end{aligned}$$

The Multiscale Method

The method is demonstrated for the term

$$\int_\Omega \nu \operatorname{curl} A \operatorname{curl} v \, d\Omega$$

using the abbreviation $\phi_z := \dfrac{\partial \phi}{\partial z}(z)$:

$$\operatorname{curl} A = \operatorname{curl} A_0 - \phi_z A_1, \qquad \operatorname{curl} v = \operatorname{curl} v_0 - \phi_z v_1$$

This results in

$$\int_\Omega \nu \operatorname{curl} A_0 \operatorname{curl} v_0 - \nu \phi_z (\operatorname{curl} A_0 v_1 + A_1 \operatorname{curl} v_0) + \nu \phi_z^2 A_1 v_1 \, d\Omega.$$

The Multiscale Method

This results in

$$\int_\Omega \nu \operatorname{curl} A_0 \operatorname{curl} v_0 - \nu\phi_z(\operatorname{curl} A_0 v_1 + A_1 \operatorname{curl} v_0) + \nu\phi_z^2 A_1 v_1 \, d\Omega.$$

The coefficients are averaged over one period P consisting of one iron sheet and one air gap:

$$\overline{\nu\phi_z} := \frac{1}{|P|} \int_P \nu\phi_z \, dz$$

For the implementations, these integrals are precomputed.

Multiscale Method in the Linear Case

Complete system (coefficients with an average of 0 are ignored):

$$
\begin{aligned}
&\int_\Omega \overline{\nu}\,\mathrm{curl}\,A_0\,\mathrm{curl}\,v_0 - \overline{\nu\phi_z}(\mathrm{curl}\,A_0 v_1 + A_1\,\mathrm{curl}\,v_0) + \overline{\nu\phi_z^2} A_1 v_1\, d\Omega \\
&+\frac{\partial}{\partial t}\int_\Omega \overline{\sigma} A_0 v_0 + \overline{\sigma\phi_z}(A_0 q + w v_0) + \overline{\sigma\phi_z^2} w q\, d\Omega \\
&+\frac{\partial}{\partial t}\int_\Omega \overline{\sigma\phi^2}\left(A_1 v_1 + \nabla w \nabla q + A_1 \frac{\partial q}{\partial r} + \frac{\partial w}{\partial r} v_1\right) d\Omega - i\int_{\Gamma(\Omega)} \tau v_0\, d\Gamma = 0 \\
&iRj - \frac{\partial}{\partial t}\int_{\Gamma(\Omega)} A_0 \tau j\, d\Gamma = uj
\end{aligned}
$$

Blue coefficients are averaged over one period P.

Multiscale Method in the Nonlinear Case

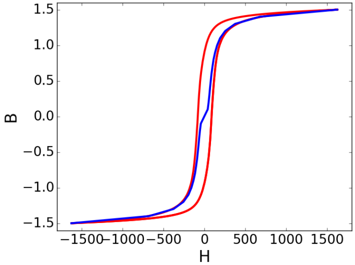
Complete system (coefficients with an average of 0 are ignored):

$$\int_\Omega \overline{\nu(A)} \operatorname{curl} A_0 \operatorname{curl} v_0 - \overline{\nu(A)\phi_z}(\operatorname{curl} A_0 v_1 + A_1 \operatorname{curl} v_0) + \overline{\nu(A)\phi_z^2} A_1 v_1 \, d\Omega$$
$$+\frac{\partial}{\partial t}\int_\Omega \overline{\sigma} A_0 v_0 + \overline{\sigma\phi_z}(A_0 q + w v_0) + \overline{\sigma\phi_z^2} w q \, d\Omega$$
$$+\frac{\partial}{\partial t}\int_\Omega \overline{\sigma\phi^2}\left(A_1 v_1 + \nabla w \nabla q + A_1 \frac{\partial q}{\partial r} + \frac{\partial w}{\partial r} v_1\right) d\Omega - i\int_{\Gamma(\Omega)} \tau v_0 \, d\Gamma = 0$$
$$iRj - \frac{\partial}{\partial t}\int_{\Gamma(\Omega)} A_0 \tau j \, d\Gamma = uj$$

How to treat red coefficients?

Treating the Nonlinearity

The time derivative is discretized using time stepping with an implicit Euler method.
In each time step a Newton iteration is performed to solve the system.



Measurement data of BH-Curve and major hysteresis loop

- $\nu = \dfrac{H}{B}$
- $\partial\nu = \dfrac{\partial H}{\partial B}$ (for Newton)
- $B = \operatorname{curl} A$
- ν is constant in air

Treating the Nonlinearity

$$\operatorname{curl} A = \operatorname{curl} A_0 - \phi_z A_1$$

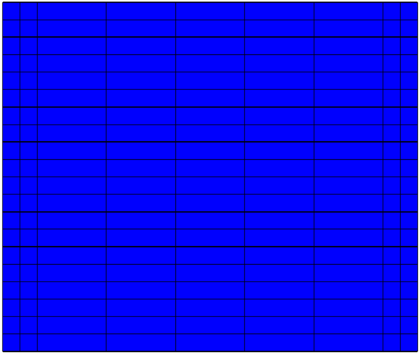
- Evaluate $\operatorname{curl} A$ at each integration point, take ϕ_z in iron.
- Use BH-curve or hysteresis model to get $\nu = \frac{H}{B}$ and $\nu_{diff} = \frac{\partial H}{\partial B}$.
- Take the average over P, using the calculated ν in iron.
- Use the result as the coefficient in this integration point.
- Achieve convergence of Newton method using damping.

The hysteresis model

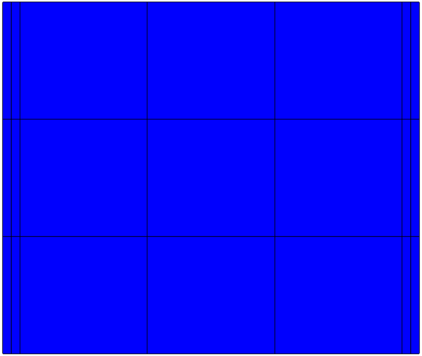
Basic principle of the used Preisach model:

- Chose a parametrized analytic ansatz for the Everett function.
- Chose the paramters by curve fitting to the limiting hysteresis loop.
- For each integration point store the past extrema.
- We calculate H from $B \Rightarrow$ use an inverse model.

Comparing the Meshes

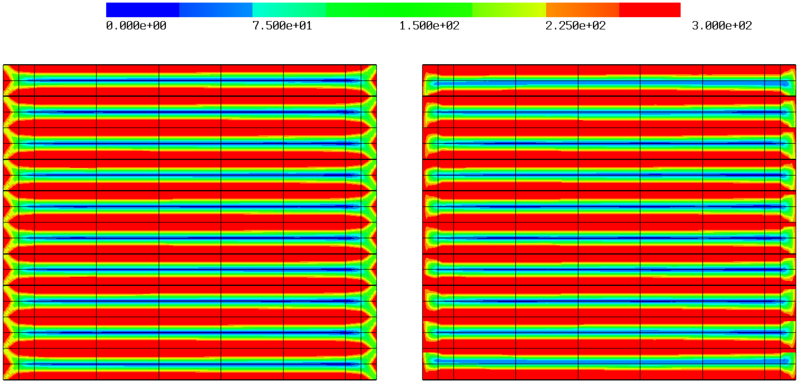


Mesh for reference solution
279 elements
2313 DOFs



Mesh for multiscale
21 elements
326 DOFs

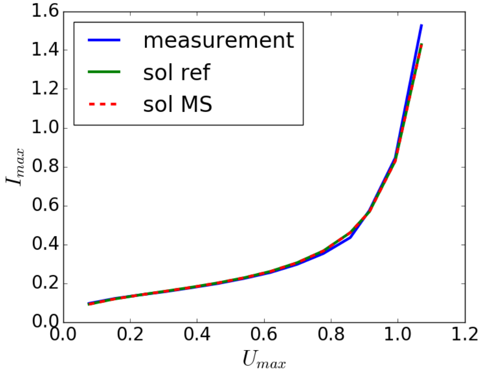
Numerical Results



$|J|$

Reference solution (left) and multiscale solution (right)

Numeric Results

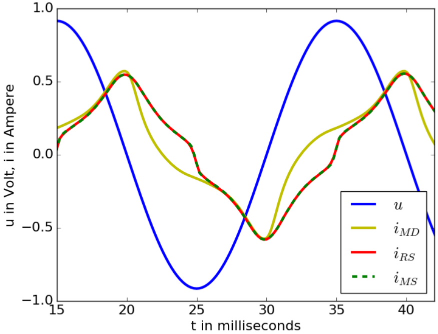


Maximum current for different voltages using only the BH-Curve

- MS solution and reference solution virtually identical
- very good agreement with measurement data

Numeric Results

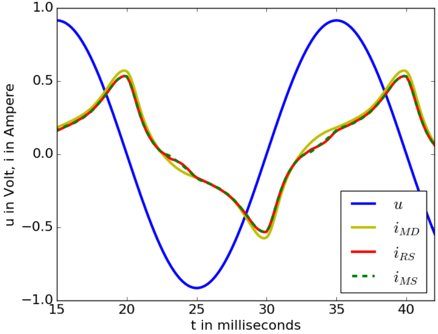
With BH-curve:



Comparison over one period
at a frequency of 50 Hz
$U_{max} = 0.923\ V$

- MS solution and reference solution virtually identical
- good agreement for minima, maxima and average compared to measurement data
- detailed behavior not reproduced correctly when using only the BH-Curve

With hysteresis:



Comparison over one period
at a frequency of 50 Hz
$U_{max} = 0.923\ V$

- MS solution and reference solution virtually identical
- good agreement for minima, maxima and average compared to measurement data
- using an approximated Preisach model, the simulation quality can be improved significantly

Conclusion

- The A-formulation of the eddy current problem with network coupling has been solved for a toroidal transformer in 2D.
- A multiscale method has been presented which reduces the number of DOFs.
- The multiscale method has been shown to be compatible the Preisach model for hysteresis.
- The numerical results are in good agreement with measurement data.

Outlook

- comparison to using the T-formulation
- 3D implementation
- higher order multiscale ansatz
- multiharmonic approach
- anisotropic material

Thank you for your attention!